



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

EARTHQUAKE RESISTANT DESIGN OF FOUNDATIONS:

Lecture 20: Dynamic Analysis of Piles under Horizontal Vibration – Continue with Numerical Example

B.K. MAHESHWARI

Professor, Dept. of Earthquake Engineering, IIT Roorkee

E-mail: bk.maheshwari@eq.iitr.ac.in



ERD of Foundations - Modules

1. Introduction
2. Shallow Foundations
- 3. Pile Foundations**
4. Well Foundations
5. SSI for Deep Foundations



Module - 3

Pile Foundations



Module -3: Chapters

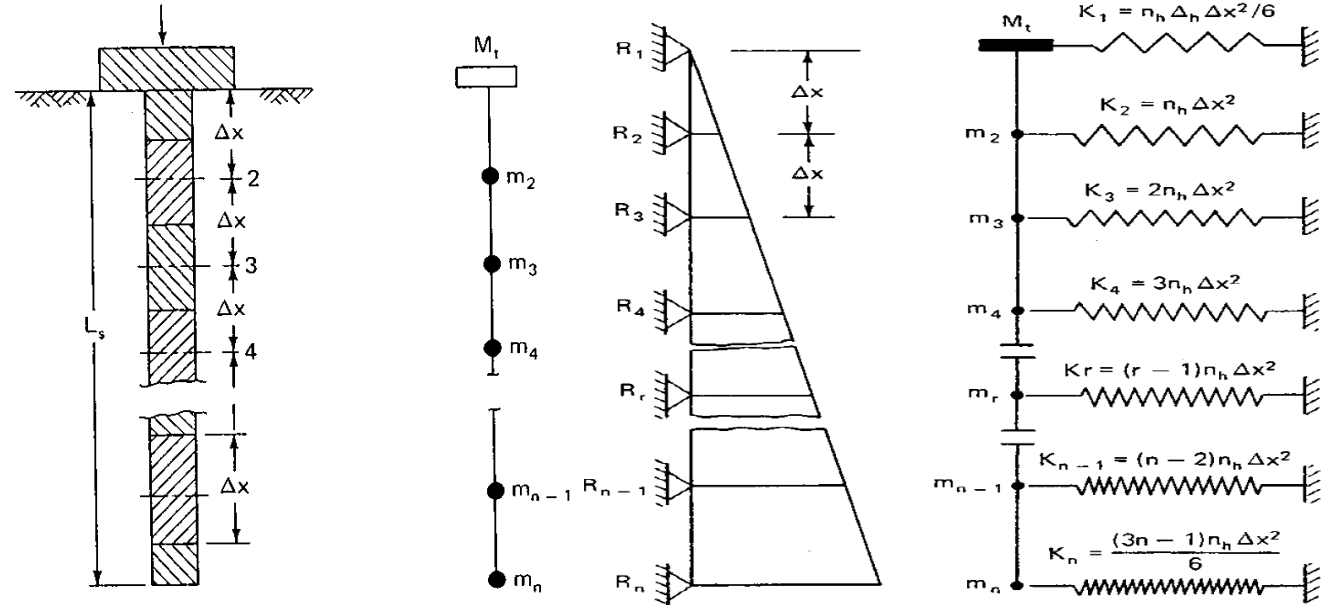
1. Piles for Static Vertical Load
2. Lateral Loads on Piles
3. Dynamic Analysis of Piles
4. Design for Lateral and Dynamic Loads
5. Codal Provisions

Chapter 1: Dynamic Analysis of Pile Foundation

- **Approaches**
- **Piles under Vertical Vibration**
- **Piles under Lateral Load**
 - Soil-Pile System
 - Natural Frequency of System
 - Method of Aseismic Analysis and Design of Piles
 - Numerical Example

Soil-Pile System (Continued...)

Assuming Soil Modulus linearly varying with depth



Natural Frequency of System

Based on the parametric analysis, nondimensional frequency factors have been obtained with respect to the basic soil parameters. The variables constituting F_{CL1} , the non dimensional frequency factor for piles embedded in soils in which the soil modulus remains constant with depth, is given as:

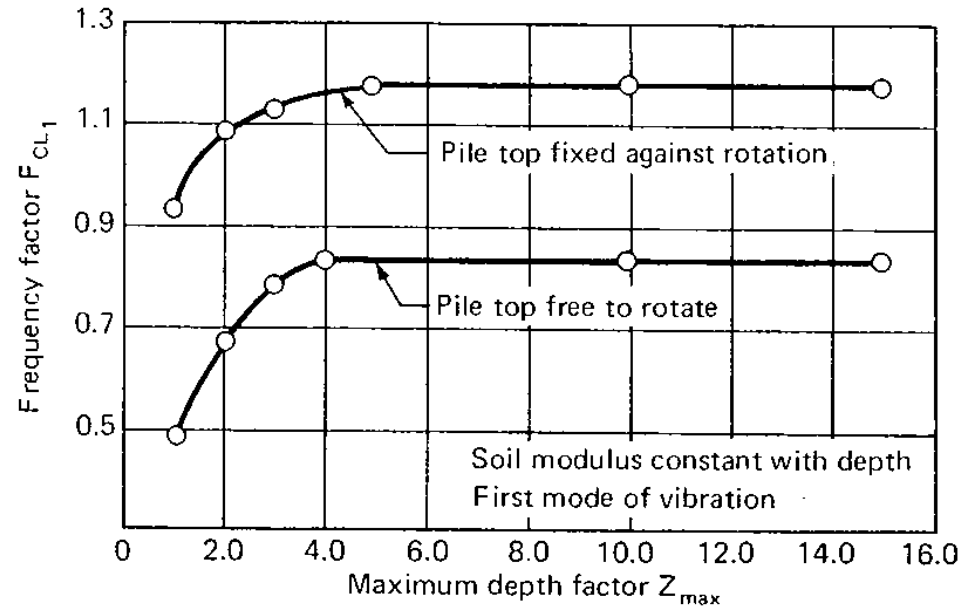
$$F_{CL1} = \omega_{n1} \sqrt{\frac{W}{gkR}}$$

In which ω_{n1} is the first natural angular frequency in radian per second, $\frac{W}{g}$ the lumped mass at the top of pile, k the soil modulus , and R , the relative stiffness factor, defined as follows:

$$R = \sqrt[4]{\frac{EI}{k}}$$

Natural Frequency of System (Continued...)

In Figure, the variation of frequency factor F_{CL1} with Z_{max} has been plotted, in which $Z_{max} = L_s/R$. It may be noted that for the given soil with pile characteristics such as flexural stiffness EI , soil stiffness in terms of k , sustained vertical load, W and Z_{max} unique frequency factor value exist.



Natural Frequency of System (Continued...)

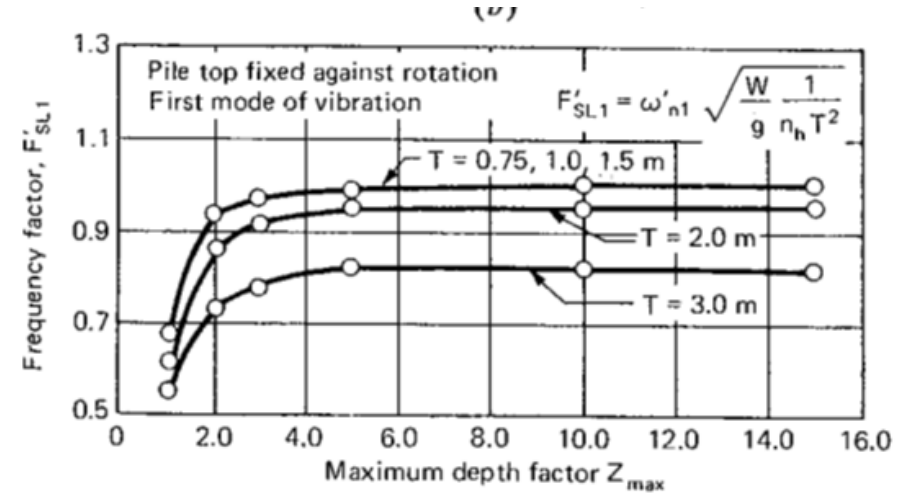
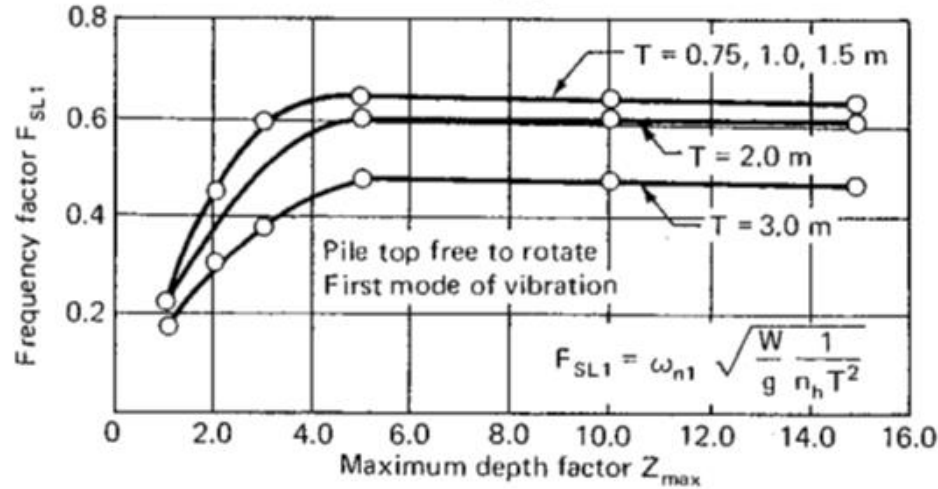
Similarly in Figure on next page, frequency factors F_{SL1} and F'_{SL1} for soils whose moduli vary linearly with depth have also been plotted for cases where the pile top is free to rotate and the pile top is restrained against rotation, respectively. The definitions of F_{SL1} and F'_{SL1} for the pile tops free to rotate and the pile top restrained against rotation are identical and given as:

$$F_{SL1} \text{ or } F'_{SL1} = \omega_{n1} \sqrt{\frac{W}{g} \frac{1}{\eta_h T^2}}$$

In which η_h is the constant of horizontal subgrade reaction for $k_x = \eta_h x$

$$T = \sqrt[5]{\frac{EI}{\eta_h}}$$

Natural Frequency of System (Continued...)

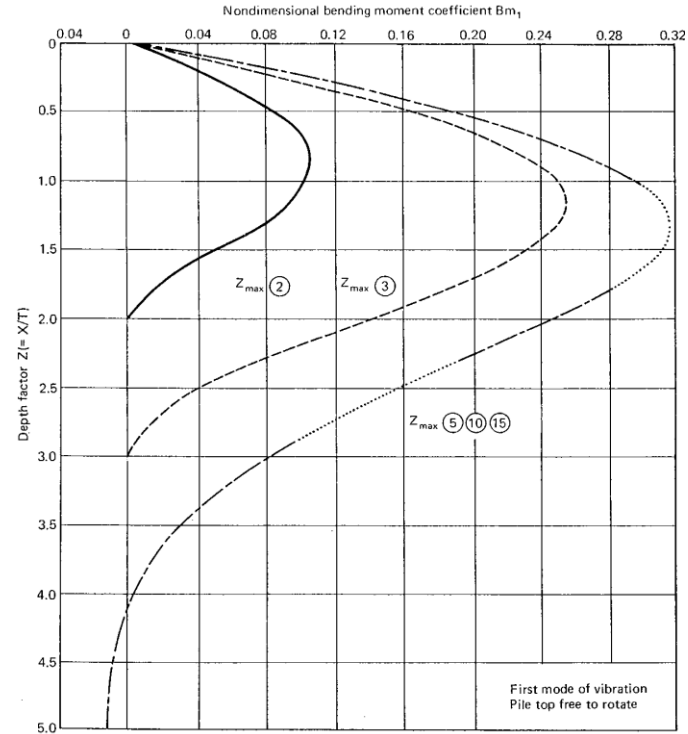
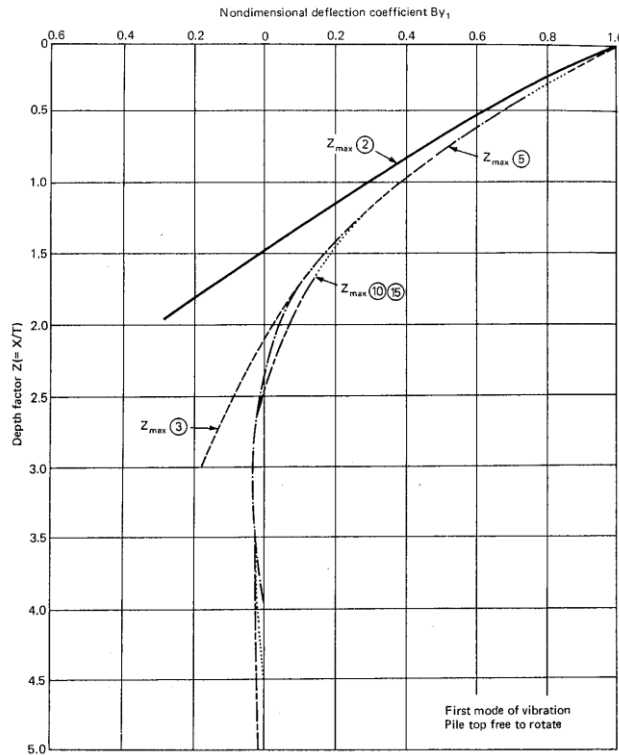


Natural Frequency of System (Continued...)

Figure shows a plot of nondimensional displacement with depth factor $z (= x/T)$ in the first mode of vibration when the pile top is restrained against rotation but is free to translate and is embedded in a soil with modulus proportional with depth. Similarly, the variation of the nondimensional bending moment coefficient with the depth of the pile in the first mode of vibrations is shown in another Figure.

It can be seen from bending moment coefficient that for a pile with $Z_{max} \geq 5$, the maximum negative bending moment coefficient $B_{me1} = 0.315$ occurs at $x/T = 1.30$. These data are sufficient for the design of piles, and the entire curve is not needed.

Natural Frequency of System (Continued...)



Aseismic Analysis and Design of Piles

Based on the dynamic analysis and the concept of the response spectrum, the following method of analysis and design of piles against earthquakes may be used. For this analysis, the following data must be obtained first

1. Soil characteristics, and boring logs of the site
2. Pile characteristics, size, EI , length, and type of pile
3. Lateral load deflection of the pile under static conditions for estimation of k or η_h

Aseismic Design of Piles (Continued...)

Design Steps

1. Estimate the dynamic soil modulus k or η_h based on the principles discussed earlier. In the absence of realistic data, the values from a static lateral load test may be modified based on engineering judgment.
2. Compute the relative stiffness factor R or T .
3. Calculate the maximum depth factor Z_{max} for a pile; Z_{max} in most practical cases will be greater than 5.
4. For the computed value of the maximum depth factor and the pile end condition, read the frequency factor.

Aseismic Design of Piles (Continued...)

5. Estimate the dead load on the pile. The mass at the pile top which may be considered vibrating with the piles is only a fraction of this load.
6. Determine the natural frequency ω_{n1} and time period in first mode of vibrations using the charts discussed, as follows:

(a) Soil modulus constant with depth:

$$F_{CL1} = \omega_{n1} \sqrt{\frac{W}{gkR}}$$

(b) Soil modulus proportional to depth:

$$F_{SL1} \text{ or } F'_{SL1} = \omega_{n1} \sqrt{\frac{W}{g} \frac{1}{\eta_h T^2}}$$

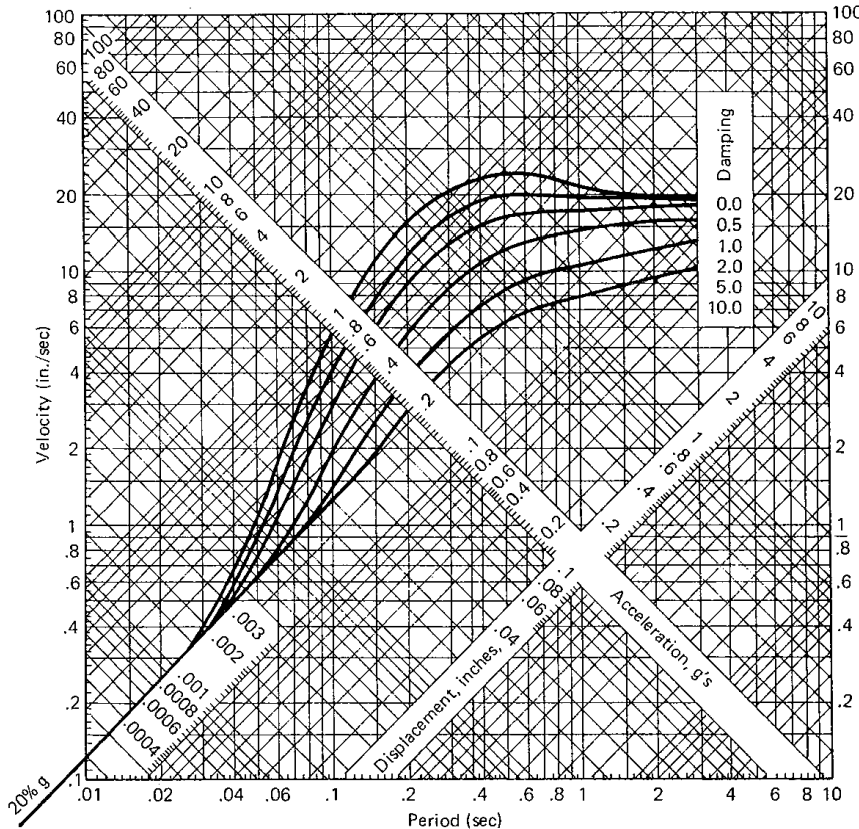
Aseismic Design of Piles (Continued...)

7. Compute the time period $T_{n1} = 2\pi/\omega_{n1}$.
8. For the computed time period, determine the spectral displacement S_d for assumed damping from Figure (as given in next slide). This is the maximum displacement of the pile head. If an accelerogram for a site has been selected, spectral response is determined for this ground motion. For the soil pile system, 5 to 10 percent damping may be assumed (Prakash, 1981).
9. Estimate the maximum bending moment in the pile section.
 - (a) soil modulus constant

$$\text{Bending moment} = A_{me} \times KR^2 \times S_d$$

The maximum value of A_{me} are given in Table

Aseismic Design of Piles (Continued...)



Maximum depth factor $Z_{\max}$	Coefficient A_{me}		
	Pile top free to rotate	Pile fixed at top against rotation	
		- ive	+ ive
2	0.13	0.9	0
3	0.24	0.9	0.04
5-15	0.32	0.9	0.18

Aseismic Design of Piles (Continued...)

(b) Soil modulus increase linearly with depth.

$$\text{Bending moment} = B_{me} \times \eta_h T^3 \times S_d$$

The maximum value of B_{me} are given in Table

Maximum depth factor $Z_{\max}$	Coefficient B_{me}		
	Pile top free to rotate	Pile fixed at top against rotation	
		– ive	+ ive
2	0.100	0.93	0
3	0.255	0.93	0.10
5–15	0.315	0.90	0.28

The pile section need to be checked against moment

Aseismic Design of Piles (Continued...)

10. For the computed maximum ground displacement, the displacement all along the length of the pile may be determined by assuming that the deflected shape in vibrations is similar to one under static conditions. For soil modulus constant with depth or soil modulus linearly varying with depth, the solutions of Davisson and Gill (1963) and Reese and Matlock (1956), respectively may be used for two cases of soil modulus. The soil reaction is then computed all along the pile lengths as follows:

(a) For soil modulus constant with depth:

$$p_x = k \cdot y_x$$

(b) For soil modulus linearly varying with depth:

$$p_x = \eta_h \cdot x \cdot y_x$$

Numerical Example- 1

A pile carries a vertical load of 77.6 t and the base shear on the of the pile cap is 3 t. If the soil is sandy in character with $\phi = 30$ and $\eta_h = 1.612 \text{ kg/cm}^3$

Under dynamic loading determine the

1. Maximum displacement of the pile head
2. Maximum bending moment in the pile
3. Soil reaction diagram along the length of pile

Assume that EI of the pile = $3.8 \times 10^{10} \text{ kg.cm}^2$, that the diameter of the pile = 30 cm, and the length of pile groups = 12.2 cm

Numerical Example- 1 (Continued...)

Solution

Step 1

$$\eta_h = 1.612 \text{ kg/cm}^3$$

Step 2

$$T = \sqrt[5]{\frac{EI}{\eta_h}} = \sqrt[5]{\frac{3.8 \times 10^{10}}{1.612}} = 118.7 \text{ cm} = 1.187 \text{ m}$$

Step 3

$$Z_{max} = 12.2/1.187 = 10.27$$

Because $Z_{max} > 5$, it is a “long” pile

Numerical Example- 1 (Continued...)

Step 4

From Fig, for $Z_{\max} > 5$ and $T = 1.187$ m,

$F_{SL1} = 0.65$ for free pile head

$F'_{SL1} = 1.00$ for pile head restrained against

rotation

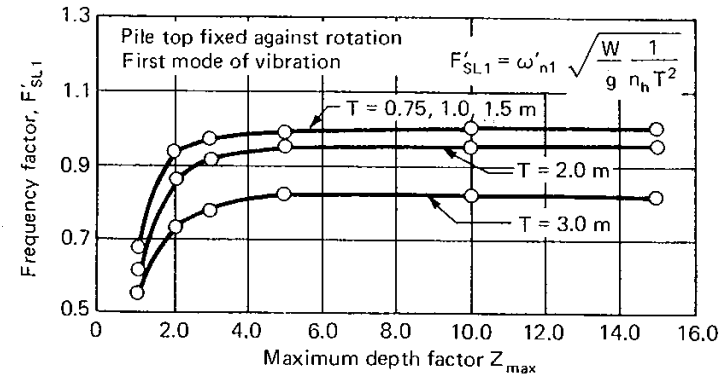
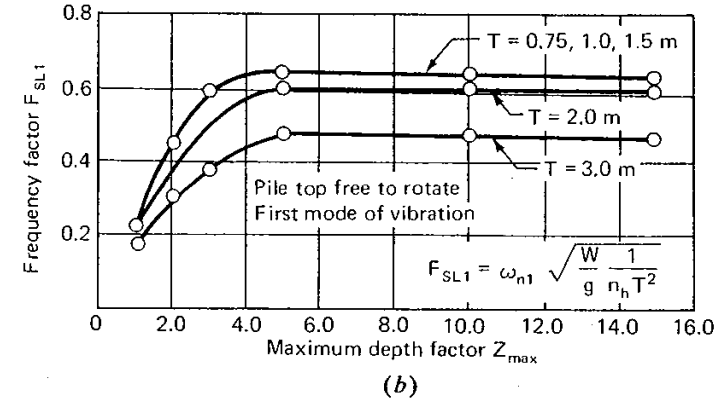
Step 5

Mass

$$M_t = 77.6 * \frac{1 \text{ t/s}^2}{981 \text{ cm}}$$

Step 6

$$\begin{aligned} \omega_{n1} &= F_{SL1} \div \sqrt{\frac{W}{g} \frac{1}{\eta_h T^2}} \\ &= 0.65 * \sqrt{\frac{981}{77.6} * \frac{1.612}{1000} * (118.72)^2} \\ &= 11.014 \text{ rad/s} \\ f_{n1} &= 11.014 / 2\pi = 1.753 \text{ Hz} \end{aligned}$$



Numerical Example- 1 (Continued...)

Step 7

$$T_{n1} = \frac{2\pi}{\omega_{n1}} = \frac{1}{1.753} = 0.570 \text{ s}$$

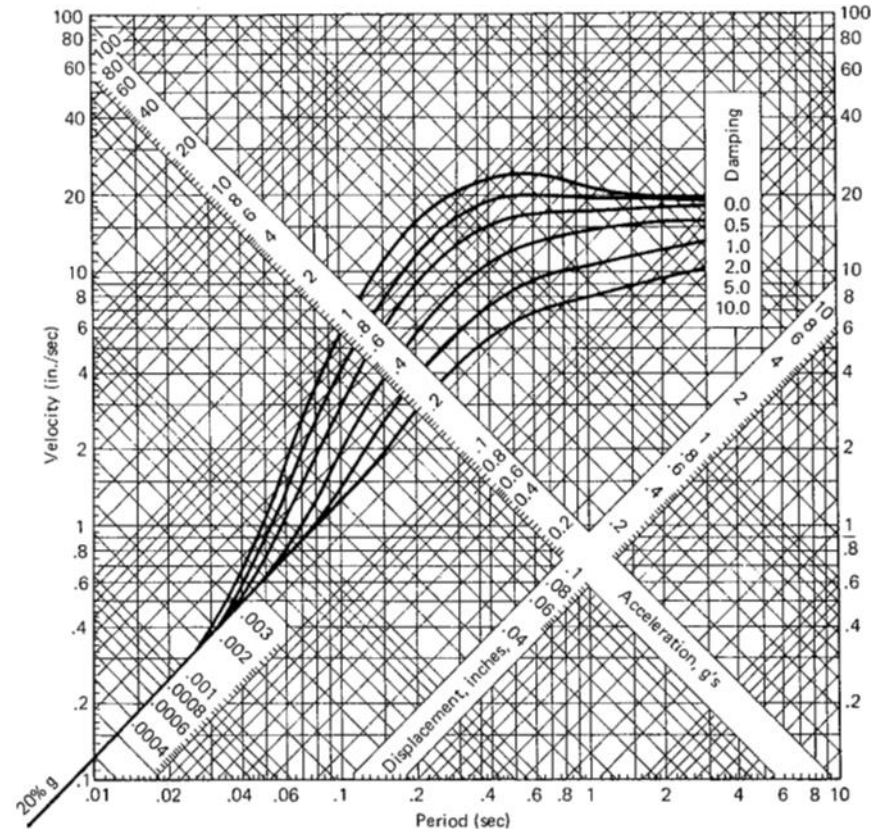
Step 8

From Fig. for $T_{n1} = 0.570 \text{ s}$, $\xi = 5\%$ spectral displacement $S_d = 0.8 \text{ in (2 cm)}$

The maximum deflection of the pile top is 2 cm.

Step 9 Maximum bending moment

$$\begin{aligned} &= B_{me} \times \eta_h \times T^3 \times S_d \\ &= 0.315 \times \frac{1.612}{1000} \times 118.7^3 \times 2 \\ &= 1698 \text{ t.cm} \end{aligned}$$



Numerical Example- 1 (Continued...)

Step 10

$$y_g = A_y \frac{Q_g T^3}{EI} = A_y B(\text{Constant})$$

For the soils with modulus increasing linearly with depth, $A_y = 2.435$

Therefore, constant in above equation is $\frac{2}{2.435} = 0.821$

Therefore, $y_x = 0.821 * A_y$,

The solution of deflection and soil reaction along the pile length is presented in Tabular form

Numerical Example- 1 (Continued...)

Step 10

The soil reaction is given by

$$p_a = ybx \frac{1+\sin\phi}{1-\sin\phi} = 162 \text{ x g/cm}$$

Numerical Example- 1 (Continued...)

$x,$ cm	Z	A_y	$y_x = 0.821 A_y,$ cm	$n_h x$ kg/cm ²	$p = n_h xy,$ kg/cm
0	0	2.435	2.00	0	0
100	0.84	1.164	0.955	161.2	153.94
200	1.68	0.327	0.268	322.4	86.40
300	2.52	0.029	0.024	483.6	11.61
400	3.36	- 0.066	- 0.054	644.8	- 34.82
500	4.21	- 0.042	- 0.034	806.0	- 27.40
600	5.05	- 0.009	- 0.007	967.2	- 6.77

References

1. **Prakash S. (1981), "Soil Dynamics", McGraw-Hill Company, New York**
2. **Prakash S. and Sharma H.D., "Pile Foundations under Dynamic Loads",**

Thank You

